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# Dynamic optimization of a dead-end filtration trajectory: Blocking filtration laws

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## Abstract

An operating model for dead-end membrane filtration is proposed based on the well-known blocking laws. The resulting model contains three parameters representing, the operating strategy, the fouling mechanism and the fouling potential of the feed. The optimal control strategy is determined by minimizing the energy consumption for a fixed final time and produced volume.

It was found that constant power filtration leads to minimal energy consumption. Constant flux and constant pressure filtration have equal energy costs. However, compared to strategies with a non-decreasing pressure and non-increasing flux, the relative savings are small. Only if the fouling mechanism resembles standard blocking and the fouling resistance is large compared to the membrane resistance, it may be attractive to implement the optimal trajectory.

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**Keywords:** Cake filtration; Complete blocking; Intermediate blocking; Standard blocking; Dynamic optimization

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## 1. Introduction

Dead-end membrane filtration is a promising technology in the field of water purification. This is due to its high selectivity, economic scalability and low chemical composition. Furthermore, compared to crossflow filtration it has low capital and energy costs.

However, membrane performance is often limited by fouling phenomena. Deposition of particulate matter from the filtered liquid causes an increase in the hydraulic resistance of the membrane system. Hence, the effort needed increases as the filtration progresses. This raises the operating costs, due to extra energy consumption and the necessity of periodic cleaning.

Several approaches to reduce the effect of membrane fouling are possible, for example: alternate process design or development of low fouling membrane materials. In this study, optimization of operating conditions is considered as a means to minimize the negative effects of fouling. One of the aspects of the operating costs is the energy consumption during the pro-

duction phase. It is investigated whether this can be lowered by application of a suitable control strategy.

In dead-end filtration, there are currently two common control strategies. In industrial applications, the constant flux strategy is usually applied. Since the produced volume is a main operating goal, this is a natural choice. For laboratory scale installations mostly constant pressure control is chosen, because this is easy to implement on a small scale. However, these are not necessarily the most efficient operating strategies. To find the optimal operating strategy, dynamic optimization is required.

Little has been published about dynamic optimization of filtration trajectories. van Boxtel et al. applied dynamic optimization to reverse osmosis of cheese whey [7]. For a single stage process, it was found that minimal operating costs can be achieved by application of a suitable crossflow velocity profile. The result is determined by a trade-off between the energy needed for the pump and the energy needed to permeate through the fouling layer. Hence, the crossflow velocity is high initially to restrict fouling; if initial fouling occurs the effect will last the entire production phase. Gradually the crossflow velocity declines to zero, because towards the end of the operation cycle the fouling rate becomes less important and the pump energy consumption may be reduced.

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Dynamic optimization requires a performance index and a process model. The performance index is in this case the energy consumption. The blocking laws are used to describe the relation between transmembrane pressure, the flux and the fouling state. Originally these models are used to describe flux decline in constant pressure filtration, under four different assumptions: cake filtration, complete blocking, intermediate blocking and standard blocking.

These models can be used for several purposes. In blocking law analysis, the fouling mechanism is inferred from the *shape* of the filtration curve. The assumption which leads to the best fit is an indication of the fouling mechanism [2–4]. Under the assumption of a certain mechanism, the *steepness* of the filtration curve is an indication of the amount of foulants in the water. Hence, this may be regarded as a means to monitor water quality. The assumption of cake filtration, for example, leads to the modified fouling index (MFI) [8]. Finally, the ability to predict a filtration curve, enables us to perform a static optimization [5,6].

The simplicity of these models has some advantages. Firstly, the model parameters are obtained easily from operating or experimental data. Secondly, the system can be solved analytically, which allows an explicit formulation of the optimal strategy. This strategy could serve as a starting point for non-ideal situations.

## 2. Theory

### 2.1. Blocking laws

In this study, the blocking laws are used as a resistance model. Hermia shows how these can be derived [1]. The author describes the total resistance as a function of the filtrated volume or the filtration state, which is defined by:

$$\frac{dw}{dt} = J \quad (1)$$

in which  $J$  is the filtration flux and  $w$  is the fouling state. Four mechanisms can be identified (see Fig. 1):

- (A) *Cake filtration*. Ideal cake filtration is based on the assumption that all particles are accumulated in a cake layer. Furthermore, it is assumed that the cake resistance is proportional to the thickness of the cake.
- (B) *Intermediate blocking*. In the intermediate blocking law, particles are allowed to settle on previously deposited particles. It is assumed that each location has an equal probability of being occupied. This means that the chance that a particle settles on a free site is equal to the ratio of free

Table 1  
Summary of the blocking laws

Mechanism	Resistance equation	$C$	$m$
(A) Cake filtration	$R(w) = R_M \left(1 + \frac{w}{w_R}\right)$	$\frac{R_M}{w_R}$	0
(B) Intermediate blocking	$R(w) = R_M e^{w/w_A}$	$\frac{1}{w_A}$	1
(C) Standard blocking	$R(w) = R_M \left(1 - \frac{w}{w_V}\right)^{-2}$	$\frac{2}{w_V R_M^{1/2}}$	3/2
(D) Complete blocking	$R(w) = R_M \left(1 - \frac{w}{w_A}\right)^{-1}$	$\frac{1}{w_A R_M}$	2

and occupied sites. It is assumed that blocked pores are impermeable.

- (C) *Standard blocking*. The standard blocking law is based on the assumption that all particles settle inside the pores. Hence, the occupied pore volume is proportional to the filtrated volume. The Hagen-Poiseuille equation is used to relate the pore volume (diameter) to the resistance.
- (D) *Complete blocking*. It is assumed that each filtrated particle participates in blocking the membrane. Hence, the blocked area depends linearly on the filtrated volume. Furthermore, it is assumed that blocked parts of the membrane are impermeable. Consequently, the resistance is inversely proportional to the fraction of free pores.

Table 1 shows the resulting resistance  $R$  as a function of the filtration state  $w$ , the membrane resistance  $R_M$  and the fouling potential of the feed:

- $w_R$  represents the specific cake resistance and is defined as the volume of feed water per unit area for which the cake resistance is equal to the membrane resistance.
- $w_A$  represents the pore blocking potential and is defined as the volume of feed water per unit area that contains enough particles to block the pores completely.
- $w_V$  represents the pore filling potential and is defined as the amount of feed water per unit area that contains enough particles to fill the pores completely.

When the resistance is differentiated with respect to the state, the four equations can be written in a common form, given in Eq. (2). The values for the constant  $C$  and the exponent  $m$  are shown in Table 1.

$$\frac{dR}{dw} = CR^m \quad (2)$$

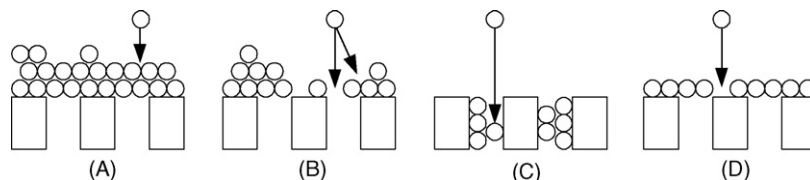


Fig. 1. Filtration laws: (A) cake filtration; (B) intermediate blocking; (C) standard blocking; (D) complete blocking.

The exponent  $m$  is defined by the fouling mechanism, resulting in values of 0, 1, 3/2 and 2 for cake filtration, intermediate blocking, standard blocking and complete blocking respectively. In principle any other real value of  $m$  can be allowed, however, in that case there is no physical interpretation. The constant  $C$  can be interpreted as a scaling factor, which is proportional to the concentration of foulants. The parameters can easily be obtained from filtration data; plotting  $\ln(dR/dw)$  versus  $\ln(R)$  results in a straight line with slope  $m$  and intercept  $\ln(C)$ .

## 2.2. Operating strategy

During dead end filtration an increase in the resistance is inevitable. The effect of the increased resistance must be distributed between pressure increase and flux decline. This distribution would be the operating strategy. The resistance is defined by Darcy's law:

$$R = \frac{\Delta P}{\eta J} \quad (3)$$

When Darcy's law is differentiated with respect to time, the relationship between the increase of the resistance, flux decline and pressure increase becomes more clear:

$$\frac{dR}{dt} = \frac{1}{\eta J} \frac{d\Delta P}{dt} - \frac{\Delta P}{\eta J^2} \frac{dJ}{dt} \quad (4)$$

Next a parameter  $s$  will be defined, which represents how the effect of an increasing resistance is distributed between a pressure incline and a flux decline. Hence, this parameter can be interpreted as the operating strategy parameter.

$$s = \frac{-(1/J)(dJ/dt)}{(1/\Delta P)(d\Delta P/dt) - (1/J)(dJ/dt)} \quad (5)$$

With this definition, it can be seen that Eq. (4) is satisfied if:

$$\frac{dJ}{dt} = -s \frac{J}{R} \frac{dR}{dt}, \quad \frac{d\Delta P}{dt} = (1-s) \frac{\Delta P}{R} \frac{dR}{dt} \quad (6)$$

If  $s$  is taken to be time invariant (the strategy does not change over time), a solution satisfying  $J(0) = J_0$  and  $R(0) = R_0$ , is given by:

$$J = J_0 \gamma^{-s} \quad (7)$$

$$\Delta P = \Delta P_0 \gamma^{1-s} \quad (8)$$

where  $\gamma$  is defined by Eq. (9) and can be interpreted as the relative difficulty of operation.

$$\gamma = \frac{R}{R_0} \quad (9)$$

From  $\gamma$ , the trajectories for the resistance Eq. (9), the flux Eq. (7), the pressure Eq. (8) and the power  $P = J\Delta P$  can easily be constructed. Hence, the entire system is defined when a trajectory for  $\gamma$  is known. Note that values of  $s$  equal to 0, 1 and 1/2 define constant flux, constant pressure and constant power filtration, respectively. Any other real value is allowed as well, however, there is no clear interpretation of the corresponding operating strategy.

To obtain a trajectory for  $\gamma$ , first Eqs. (1) and (2) are used to obtain a different representation for  $J$ :

$$J = \frac{1}{C} R^{-m} \frac{dR}{dt} \quad (10)$$

This result is substituted in Eq. (7). After rearranging the following equation is obtained:

$$\frac{d\gamma}{dt} = K_0 \gamma^{m-s} \quad (11)$$

with

$$K_0 = C J_0 R_0^{m-1} \quad (12)$$

After separation of variables and integration, the resulting trajectories for the relative difficulty of operation in time can be defined by:

$$\gamma(t) = \begin{cases} (1 + (s - m + 1)K_0 t)^{1/(s-m+1)}, & \text{for } s - m + 1 \neq 0 \\ e^{K_0 t}, & \text{for } s - m + 1 = 0 \end{cases} \quad (13)$$

The state trajectory may be obtained by integrating Eq. (2). It is given by:

$$w = \begin{cases} \frac{1}{C} \frac{R_0^{1-m}}{1-m} \left( \gamma^{1-m} - \left( \frac{R_M}{R_0} \right)^{1-m} \right), & \text{for } 1 - m \neq 0 \\ \frac{1}{C} \left( \ln(\gamma) - \ln \left( \frac{R_M}{R_0} \right) \right), & \text{for } 1 - m = 0 \end{cases} \quad (14)$$

### 2.2.1. Initial and final conditions

The function defined by Eqs. (12) and (13) can be used to calculate a trajectory, when the fouling parameters and the initial conditions are known. This is fine for monitoring and prediction. However, for comparison it is required that the two strategies produce the same volume in the same time interval. Hence, the initial condition is rewritten in a form that contains desired final conditions.

The produced volume and fouling state are closely related, hence the final condition can be formulated as:

$$\gamma(T) = \gamma_T \quad (15)$$

When Eq. (13) is evaluated at  $t = T$ , combined with Eq. (15) and solved for  $K_0$ , the following result is obtained:

$$K_0 = \begin{cases} \frac{1}{s - m + 1} \frac{1}{T} (\gamma_T^{s-m+1} - 1), & \text{for } s - m + 1 \neq 0 \\ \frac{1}{T} \ln(\gamma_T), & \text{for } s - m + 1 = 0 \end{cases} \quad (16)$$

Substitution leads to:

$$\gamma = \begin{cases} \left( 1 + (\gamma_T^{s-m+1} - 1) \frac{t}{T} \right)^{1/(s-m+1)}, & \text{for } s - m + 1 \neq 0 \\ \gamma_T^{t/T}, & \text{for } s - m + 1 = 0 \end{cases} \quad (17)$$

The corresponding initial flux which can be found by using Eqs. (12) and (16), can be given by:

$$J_0 = \begin{cases} \frac{R_0^{1-m}}{CT(s-m+1)}(\gamma_T^{s-m+1} - 1), & \text{for } s-m+1 \neq 0 \\ \frac{R_0^{1-m}}{CT} \ln(\gamma_T), & \text{for } s-m+1 = 0 \end{cases} \quad (18)$$

### 2.3. Optimization

Dynamic optimization is used to distribute the production rate over time to achieve minimal energy consumption. This can be done by finding a balance between producing at a high rate when filtration is easy and producing at a lower rate when filtration is difficult. The formulation of a dynamic optimization problem requires a system model, a performance index and initial and final conditions.

The system describes the change of the state variable under the influence of the control variable. Here, the difficulty is chosen as the state variable and the flux is chosen as the control variable. Several other choices would be possible, however, this does not influence the results. For the system equation, Eq. (11) can not be used, because it can not be assumed that  $s$  is time invariant for the optimal strategy. The state equation is found by combining Eqs. (1), (2) and (9):

$$\frac{d\gamma}{dt} = CR_0^{m-1}\gamma^m J \quad (19)$$

The performance index is the energy consumption per unit area  $E$ . This is equal to the integral of the power per unit area  $J\Delta P$ . In terms of the production rate and difficulty, the goal functional can be given by:

$$E = \eta R_0 \int_0^{t_e} J^2 \gamma dt \quad (20)$$

The formulation of the dynamic optimization problem is concluded by specifying the initial and final conditions. The initial value of  $\gamma$  is per definition 1. For a fixed produced volume in a fixed time interval, the final conditions are:  $t_e = T$  and  $\gamma(t_e) = \gamma_T$ .

The minimum principle will be used to find the minimal energy consumption. An introduction to this principle can be found in [9]. The Hamiltonian is defined by Eq. (21), in which the adjoined state  $\lambda$  is introduced.

$$\mathcal{H} = \lambda CR_0^{m-1}\gamma^m J + \eta R_0 J^2 \gamma \quad (21)$$

The first necessary condition for optimality requires that the control variable minimizes the Hamiltonian. Since the variables are not bounded and the flux appears quadratically, this corresponds with:

$$\frac{\partial \mathcal{H}}{\partial J} = \lambda CR_0^{m-1}\gamma^m + 2\eta R_0 J \gamma = 0 \quad (22)$$

This equation can be used to find the adjoined state:

$$\lambda = -\frac{2\eta R_0 \gamma}{CR_0^{m-1}\gamma^m} \quad (23)$$

When this is substituted into Eq. (21) it follows that the value of the Hamiltonian in the optimum is given by:

$$\mathcal{H}^* = -\eta R_0 J^2 \gamma \quad (24)$$

Hence, it turns out that the value of the Hamiltonian is proportional to the power. According to the optimal control theory, the Hamiltonian remains constant. Therefore, a constant power strategy is optimal. This corresponds to the previously described trajectories when  $s = 1/2$ .

### 2.4. Energy consumption

For both the optimal and the reference strategies, the value of the energy consumption can be found from the integral over the power trajectory. When Eq. (7) is substituted into Eq. (20), this is given by:

$$E = \eta R_0 J_0^2 \int_0^T \gamma^{1-2s} dt \quad (25)$$

Substitution of Eqs. (17) and (18) into Eq. (25) gives (for  $2-s-m \neq 0$  and  $s-m+1 \neq 0$ ):

$$E = \frac{\eta R_0^{3-2m}}{C^2 T} \left( \frac{\gamma_T^{s-m+1} - 1}{s-m+1} \right) \left( \frac{\gamma_T^{2-s-m} - 1}{2-s-m} \right) \quad (26)$$

The relative energy consumption is defined as the energy consumption of a certain strategy  $E$  divided by the optimal energy consumption  $E^*$ . Hence, this is given by Eq. (27) for  $m - (3/2) \neq 0$  and  $(3/2) - m \neq (1/2) - s$ ,  $(3/2) - m \neq s - (1/2)$ . This equation depends on three factors, the fouling mechanism  $m$ , the strategy  $s$  and the final difficulty  $\gamma_T$ .

$$\begin{aligned} \frac{E}{E^*} &= \left( \frac{\gamma_T^{((3/2)-m)-((1/2)-s)} - 1}{((3/2)-m) - ((1/2)-s)} \right) \\ &\times \left( \frac{\gamma_T^{((3/2)-m)+((1/2)-s)} - 1}{((3/2)-m) + ((1/2)-s)} \right) \left( \frac{(3/2)-m}{\gamma_T^{(3/2)-m} - 1} \right)^2 \end{aligned} \quad (27)$$

Note that this function can be made continuous in the undefined points by making use of the following standard limit, in which  $x$  can be  $(3/2) - m$  or  $((3/2) - m) \pm ((1/2) - s)$ .

$$\lim_{x \rightarrow 0} \frac{\gamma_T^x - 1}{x} = \ln(\gamma_T) \quad (28)$$

This also follows from integration of Eq. (25) under the mentioned conditions. For example, if  $(3/2) - m \rightarrow 0$  and  $s \neq (1/2)$  then Eq. (27) changes into:

$$\begin{aligned} \frac{E}{E^*} &= \left( \frac{\gamma_T^{((3/2)-m)-((1/2)-s)} - 1}{((3/2)-m) - ((1/2)-s)} \right) \\ &\times \left( \frac{\gamma_T^{((3/2)-m)+((1/2)-s)} - 1}{((3/2)-m) + ((1/2)-s)} \right) \left( \frac{1}{\ln(\gamma_T)} \right)^2 \end{aligned} \quad (29)$$

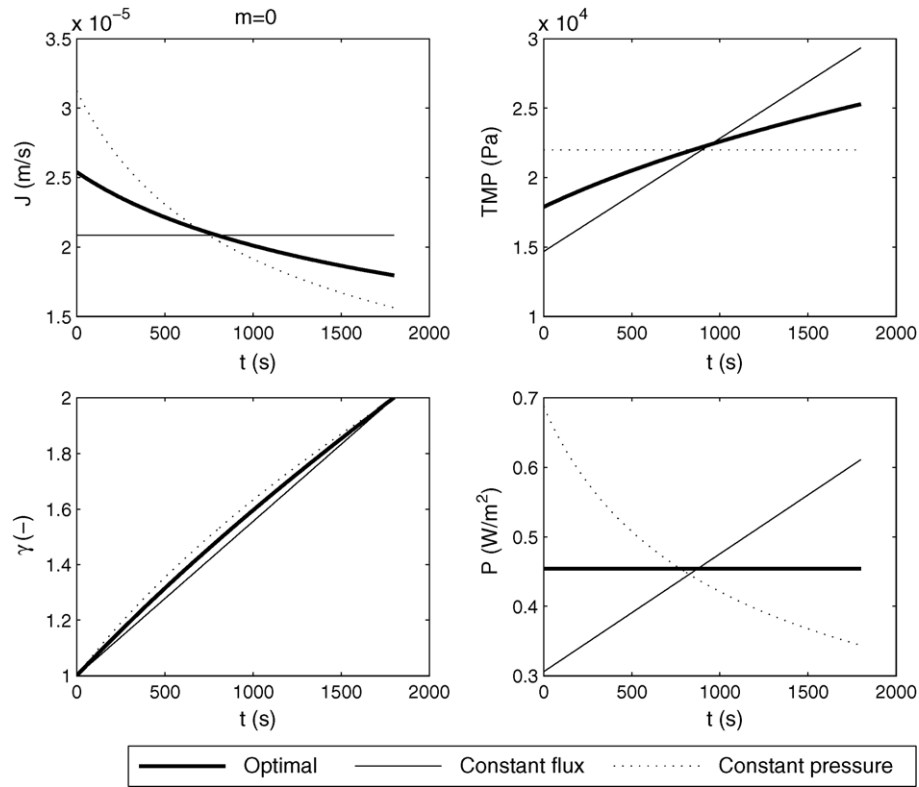


Fig. 2. Examples of filtration trajectories for ideal cake filtration ( $m = 0$ ) for constant flux, pressure and power filtration ( $\gamma_T = 2$ ,  $w_T = 0.0375$  m,  $T = 1800$  s,  $R_0 = 7 \times 10^{11}$ ).

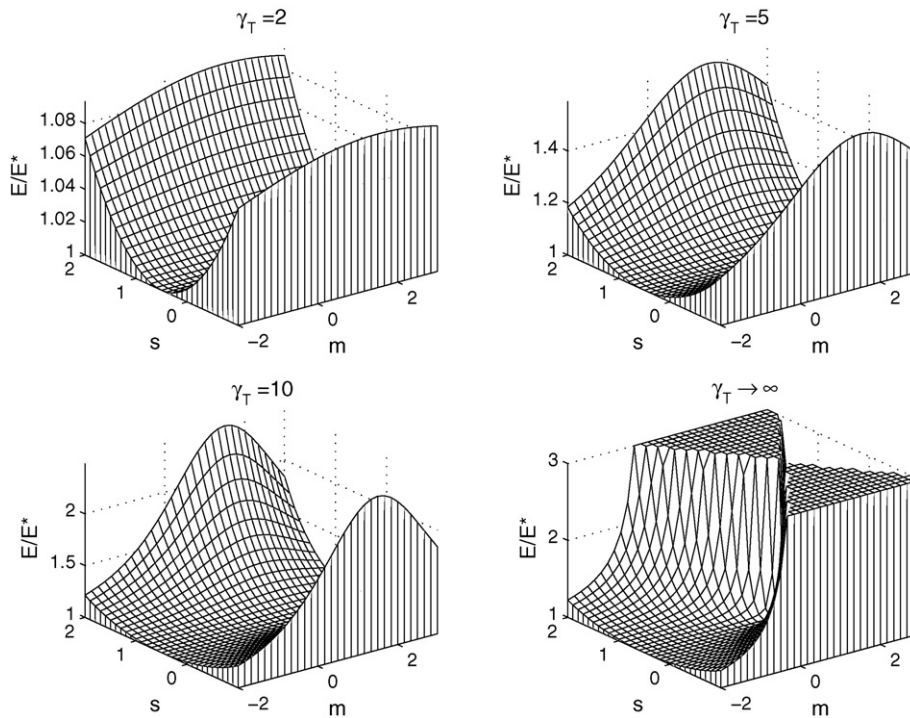


Fig. 3. Energy consumption relative to the optimal energy consumption as function of mechanism parameter  $m$ , strategy parameter  $s$  and final difficulty  $\gamma_T$ . The bottom right figure is restricted to  $E/E^* \leq 3$ .



### 3. Results

Fig. 2 shows the trajectories of the optimal (constant power) and the reference strategies (constant flux and constant pressure), calculated for cake filtration. It can be seen that the optimal trajectory results in both flux decline and a pressure increase. By distributing the increasing resistance over these two effects, the power is kept constant. Fig. 3 shows the relative energy consumption as a function of the strategy  $s$ , the fouling mechanism  $m$  for different final states  $\gamma_T$ . Some results which follow from Eq. (27) and this figure are:

- The relative energy consumption curve is symmetrical around the optimal strategy ( $s = 1/2$ ) and standard blocking ( $m = 3/2$ ). Therefore the energy consumption of constant pressure and constant flux filtration are equal.
- The maximum energy saving is achieved for the standard blocking mechanism ( $m = 3/2$ ). In the neighbourhood of this line it is more attractive to choose the operating strategy carefully.
- In the bottom right plot of Fig. 3, the potential savings are shown for a very large final difficulty  $\gamma_T \rightarrow \infty$ . The  $m$ ,  $s$ -plane can be divided into quarters around standard blocking ( $m = 3/2$ ) and the optimal strategy ( $s = 1/2$ ). In the areas in which  $|m - (3/2)| < |s - (1/2)|$  holds, the potential energy savings are limited. When for example, the fouling mechanism is cake formation ( $m = 0$ ), the difference between the optimal strategy and constant flux or pressure filtration is less than 12.5%. For constant flux and constant pressure filtration, the potential savings are limited for  $m < 1$  or  $m > 2$ . Between intermediate and complete blocking ( $1 < m < 2$ ) the potential savings scale with the difficulty of the filtration.

### 4. Conclusion

The adapted classical filtration laws are suitable for the description of filtration curves, because the model parameters are easily obtained and the model can be solved analytically for numerous operating strategies, including constant flux, constant pressure and constant power filtration.

Under ideal conditions, constant power filtration leads to a minimal energy consumption; constant flux and constant pressure filtration are equally expensive. However, the potential savings are small in the region between constant flux and constant pressure filtration. If the amount of fouling is not too large, any strategy with a decreasing or constant flux and an increasing or constant pressure is acceptable. However, if the amount of fouling is large, it may only be attractive to use the optimal strategy if the fouling mechanism is such that the potential savings scale with the difficulty of the filtration.

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### Nomenclature

$C$	fouling potential ( $\text{m}^{m-2}$ )
$E$	energy consumption ( $\text{J}/\text{m}^2$ )
$\mathcal{H}$	Hamiltonian ( $\text{W}/\text{m}^2$ )
$J$	flux ( $\text{m}/\text{s}$ )
$K_0$	fouling rate constant ( $\text{s}^{-1}$ )
$m$	mechanism parameter
$P$	power ( $\text{W}/\text{m}^2$ )
$\Delta P$	trans membrane pressure (Pa)
$R$	resistance ( $\text{m}^{-1}$ )
$R_M$	membrane resistance ( $\text{m}^{-1}$ )
$s$	strategy parameter
$t$	time (s)
$T$	final time (s)
$w$	fouling state (m)
$w_A$	pore filling potential (m)
$w_R$	cake resistance potential (m)
$w_V$	blocking potential (m)

### Greek symbols

$\gamma$	difficulty
$\eta$	viscosity (Pa s)
$\lambda$	adjointed state ( $\text{J m}^{-2}$ )

### Indices

0	initial ( $t = 0$ )
$T$	final ( $t = T$ )
*	optimal

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